$T(n) = 3T(\frac{n}{3}) + \theta(1)$ $\log_b a = 1$ $\theta(1) \begin{cases} O(n^{1-\varepsilon}) \\ \theta(n^1) \\ \Omega(n^{1+\varepsilon}) \end{cases} \Rightarrow \varepsilon = 1.$

$\uparrow a$ $\uparrow b$ $\uparrow f(n)$

Cas 1: $\theta(1) = O(n^{1-1}) = O(1) \Rightarrow T(n) = \theta(n).$

$U(n) = 2U(\frac{n}{3}) + \theta(1)$ $\log_b a = \log_3(2) =$

$\theta(1) \begin{cases} O(n^{\log_3 2 - \varepsilon}) \Rightarrow \varepsilon = \log_2 3 \\ \theta(n^{\log_3 2}) \leftarrow \text{croissant} \\ \Omega(n^{\log_3 2 + \varepsilon}) \end{cases}$

$\uparrow$ cst

Cas 2: $\theta(1) = O(n^{\log_3(2) - \log_3(2)}) = O(1)$

$U(n) = \theta(n^{\log_3(2)})$

$V(n) = V(n/2) + n + 2 \Rightarrow \{a = 1, b = 2, f(n) = n+2\}$ $\log_2 1 = 0$

Le 3e cas s'applique

$V(n) = \theta(n+2) = \theta(n)$

$a f(n/b) \leq c f(n)$

$\frac{n}{2} + 2 \leq c(n+2)$

$\leq cn + 2c$

$\Downarrow$

$\frac{1}{2} \leq c < 1$

$n+2$ $\begin{cases} O(n^{0-\varepsilon}) \leftarrow \text{decroissant} \\ \theta(n^0) \leftarrow \text{cst.} \\ \Omega(n^{0+\varepsilon}) \end{cases}$

$\uparrow$ croissant

$\varepsilon = 1.$ $n+2 = \Omega(n)$

$W(n) = W(\lfloor n/2 \rfloor) + W(\lceil n/2 \rceil) + \theta(\log n) =$

$= 2W(n/2 + O(1)) + \theta(\log n)$ $\log_2 2 = 1$

$\varepsilon = 0,9$ $\theta(\log n) = O(n^{0,1})$

$W(n) = \theta(n)$

$f(n) = \theta(\log n) = \begin{cases} O(n^{1-\varepsilon}) \\ \theta(n) \\ \Omega(n^{1+\varepsilon}) \end{cases}$

$\forall \varepsilon \in \mathbb{R} \mid 0 < \varepsilon < 1.$

$X(n) = 2X(n/2) + \theta(n\log(n))$; $\log_2 2 = 1.$

$\varepsilon > 0$:

$\lim_{n \to +\infty} \frac{n \log n}{n^{1+\varepsilon}} = \lim_{n \to +\infty} \frac{\log n}{n^{\varepsilon}} = 0$

$f(n) = \theta(n \log n) = \begin{cases} O(n^{1-\varepsilon}) \\ \theta(n) \\ \Omega(n^{1+\varepsilon}) \end{cases}$

ccl : le théorème ne s'applique pas.

3.2 : Matrix Addition

1) for $i \leftarrow 0$ to n
   for $j \leftarrow 0$ to n
      $A[i][j] = A[i][j] + B[i][j]$.

$\Theta(n^2)$ Pour remplir n^2 cases, il faut faire n^2 operation.

3.3

0
1
2] $\Theta(1)$
3
4] $\Theta(n^2)$
5
6
7
8] $2T(n/2) + \Theta(n^2)$
9 $\Theta(n^4)$

$T(n) = 8T(n/2) + \Theta(n^2)$
$T(1) = \Theta(1)$

On compare $\Theta(n^2)$ à $\begin{cases} O(n^{3-\varepsilon}) \\ \Theta(n^3) \\ \Omega(n^{3+\varepsilon}) \end{cases}$

$\varepsilon = 1$: $\Theta(n^2) = O(n^{3-1}) = O(n^2)$ On est dans le cas 1.

$T(n) = \Theta(n^3)$.

3.4

0
1
2] $\Theta(1)$
3
4] $\Theta(n^2)$
5
6
7
8
9
10
11] $T(n/2) + \Theta(n^2)$
12
13
14
15
16] $\Theta(n^2)$

$T(1) = \Theta(1)$

$T(n) = 7T(n/2) + \Theta(n^2)$: $\log_2 7$

On compare $\Theta(n^2)$ $\begin{cases} O(n^{\log_2 7-\varepsilon}) \\ \Theta(n^{\log_2 7}) \\ \Omega(n^{\log_2 7+\varepsilon}) \end{cases}$

$\log_2 7 - 2 < \varepsilon < 0$

$\Theta(n^2) = O(n^{\log_2 7 - \varepsilon})$

3.5 :

$T_{min}(p) = T_{min}(p/2) + \Theta(1)$
$= \Theta(\log p)$

$T_{max}(p) = T_{max}(\lfloor p/2 \rfloor) + \Theta(1)$
$= \Theta(\log p)$

$T(n) = \Theta(\log n)$ en cas général.

```
MatPow (X, p, n)
  if p == 0
     return I_n
  if odd(p)
     return SMUL(X, MatPow(SMUL(X, X, n), ⌊P/2⌋, n), n)
  else
     return MatPow(SMUL(X, X, n), p/2, n)
```

$T(p) = \Theta(n^{\log_2 7} \times \log p)$