### Physics Midterm n°2

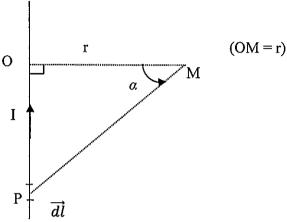
Calculators and extra-documents are not allowed. Answer only on exam sheets.

**MCQ** 

(4 points; no negative points)

#### Circle the right answer.

We are studying the magnetic field which is generated by a constant current I, which is flowing in an infinite wire oriented along the axis (Oz).



The elementary field created by  $\vec{dl}$  reads, using Biot-Savart's law:  $d\vec{B}(M) = \frac{\mu_0 I}{4\pi} \frac{dl \wedge PM}{(PM)^3}$ 

- 1- The total magnetic field and the current I are:
  - a) collinear
- b) orthogonal
- c) parallel
- 2- The norm of the vector  $\vec{dl}$  can be written:
  - a) rdα
- b) dr
- c) dz
- 3- The circulation  $C(\vec{B})$  of a magnetic field  $\vec{B}$  along a path C is defined by  $C(\vec{B}) = \int \vec{B} \cdot d\vec{l}$ . What is the correct claim?
  - a)  $C(\vec{B}) \leq 0$
- b)  $C(\vec{B}) \geq 0$
- c) in general  $C(\vec{B})$  can be any real number
- 4- The closed path  $\mathcal C$  of the question 3 encloses some ingoing currents  $I_i$  (still respecting the conventional direct orientation). Now what is the correct claim?
  - a)  $C(\vec{B}) \geq 0$
- b)  $C(\vec{B}) \leq 0$
- c) in general  $C(\vec{B})$  can be any real number
- 5- Before using Ampère's theorem, we study the symmetries of the system. If we find a symmetry plane  $\mathcal{P}$ , then:
  - a)  $\vec{B} \perp \mathcal{P}$
- b)  $\vec{B}$  and  $\mathcal{P}$  are collinear
- c)  $\vec{B} \in \mathcal{P}$

6- How do we define a field line? Locally  $\overrightarrow{dl}$  is tangent to a field line.

- a)  $\vec{B} \wedge \vec{dl} = \vec{0}$
- b)  $\vec{B} \cdot \vec{dl} = 0$
- c)  $\|\vec{B}\| = cst$

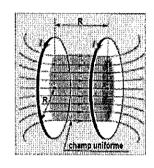
7- The magnetic field is a field with a flux:

- a) intensive
- b) extensive
- c) conservative

8- The magnetic force  $\overrightarrow{F_m}$  generated by  $\overrightarrow{B}$  on a charge q with velocity  $\overrightarrow{v}$  is always:

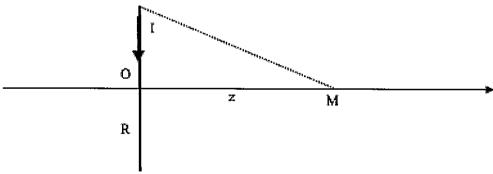
- a) collinear to  $\vec{v}$
- b) orthogonal to  $\vec{v}$
- c) resistive

Exercise 1 (8 points) Helmholtz's coils



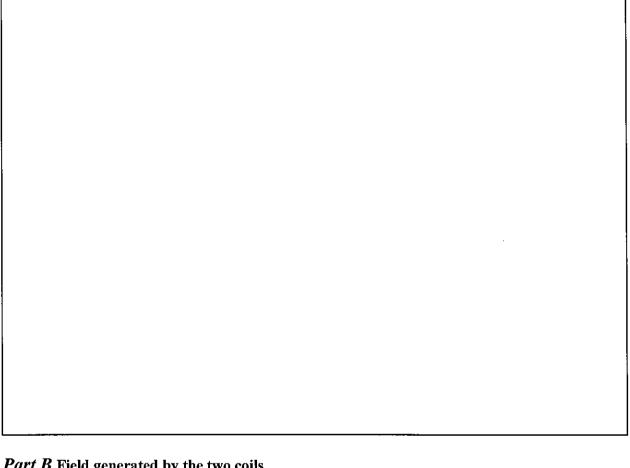
## **Part A** Field generated by a single coil

We are studying the magnetic field  $\vec{B}$  generated by a coil of radius R, of axis (Oz) and with a constant current I inside.



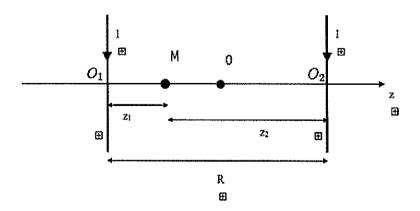
1- Study the invariances and the symmetries. Deduce the relevant components and dependences of the magnetic field  $\vec{B}$ .

2- Biot and Savart's law states that the elementary magnetic field reads  $d\vec{B}(M) = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \wedge P\vec{M}}{(PM)^3}$ , with P belonging to the coil and M on the axis (Oz). Express the magnetic field  $\vec{B}$  generated by the coil.

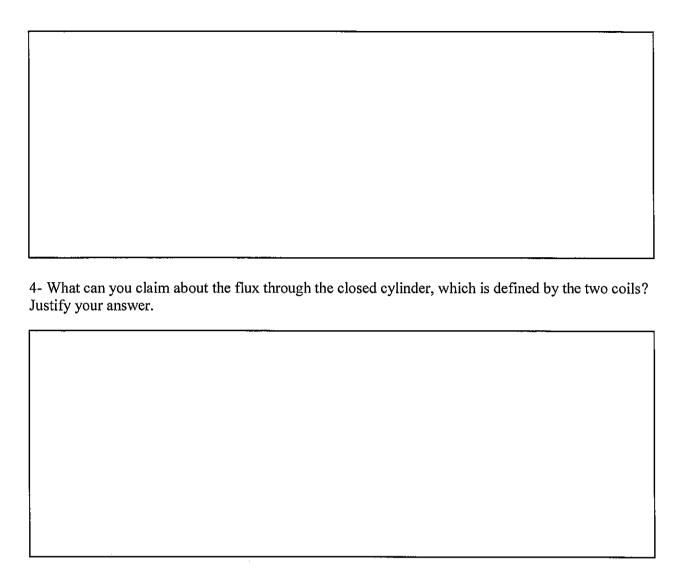


### Part B Field generated by the two coils

Both coils have the same radius, the same axis (Oz) and the distance between them is R. The drawing below does not represent the correct scale. The current flows in the same orientation in both coils. The origin of the coordinate is O, the middle between O<sub>1</sub> and O<sub>2</sub>. We now want to find the magnetic field  $\vec{B}$  at some point M of coordinate z = OM on the axis (Oz) between the two coils.

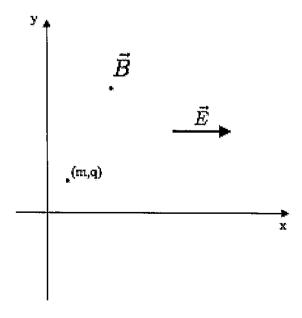


1- Denoting $z_1 = O_1 M$ and $z_2 = O_2 M$ , express in terms of z and $\frac{R}{2}$ their respective squares $z_1^2$ and $z_2^2$ .
2- Using the results of Part A and the previous question, express the total field $\vec{B}$ generated at M.
3- One recalls the Taylor expansion at first order of $x \to \frac{1}{(1+x)^n}$ , $n \in \mathbb{R}$ :
$\frac{1}{(1+x)^n} = 1 - nx$
Using this result, give the expression of $\vec{B}(z)$ for z close to 0.



# Exercise 2 (8 points)

We are studying the motion of a charge q of mass m. It is moving in two uniform fields: an electric one along (Ox), the other one is magnetic and outgoing along (Oz).



The charge $q$ can a priori move in the three space dimensions. We will denote $\vec{v}$ its velocity vector. The charge $q$ has no initial speed.
1- Recall the expression of the magnetic force $\overrightarrow{F_m}$ and that one of the electric force $\overrightarrow{F_e}$ .
2- Write the second Newton law and give the differential equations describing the motion.
3- Decompose the motion in two parts, a rotation and a translation. Describe the trajectory of the charge $q$ .