

Electrostatique

Exercice 1

$$1. V_{BDE}(A) = k \sum \frac{q_i}{r_i} = \frac{k}{r} \sum q_i = \frac{k}{a} (-3q) = -\frac{3kq}{a}$$

$$2.a \quad V(A) = V_{BDE}(A) + V_{CHP}(A) + V_C(A)$$

$$\stackrel{\text{Valeurs}}{=} -\frac{3kq}{a} + \frac{k}{a\sqrt{2}} (3q) + \left(\frac{-kq}{a\sqrt{3}} \right)$$

$$= \frac{kq}{a} \left(-3 + \frac{3}{\sqrt{2}} - \frac{1}{\sqrt{3}} \right) < 0$$

$$b \quad E_p(A) = q(A) \times V(A)$$

$$\stackrel{\text{Sous}}{=} q \times \frac{kq}{a} \left(-3 + \frac{3}{\sqrt{2}} - \frac{1}{\sqrt{3}} \right)$$

$$= \frac{kq^2}{a} \left(-3 + \frac{3}{\sqrt{2}} - \frac{1}{\sqrt{3}} \right) < 0$$

Exercice 2.



$$1. F(A) = F_{B/A} - F_{C/A}$$

$$= \frac{k \times e^2}{a^2} - \frac{k \times e^2}{(2a)^2}$$

$$= \frac{-4ke^2 - ke^2}{4a^2}$$

$$= \frac{3}{4} \frac{ke^2}{a^2}$$

$$\vec{F}(A) = \vec{F}_{B/A} + \vec{F}_{C/A}$$

$$F_{B/A} > F_{C/A}$$

$$\Rightarrow \vec{F}(A) = \frac{3}{4} \frac{ke^2}{a^2} \vec{T}$$

$$F(C) = F(A) = \frac{3}{4} \frac{ke^2}{a^2}$$

$$\vec{F}(C) = -\frac{3}{4} \frac{ke^2}{a} \vec{T}$$

↳ car $\vec{F}(C)$ est opposé à \vec{T} (vecteur unitaire)

$$\begin{aligned} \vec{F}(B) &= \vec{F}_{A/B} + \vec{F}_{C/B} \\ &= \vec{0} \end{aligned}$$

de même norme et de sens opposé

$$\vec{E}_A = \frac{\vec{F}(A)}{e} = \frac{3}{4} \frac{ke}{a^2} \vec{T}$$

$$\vec{E}_B = \frac{\vec{F}(B)}{-e} = \vec{0}$$

$$\vec{E}_C = \frac{\vec{F}(C)}{e} = -\frac{3}{4} \frac{ke}{a^2} \vec{T}$$

$$2. V(A) = k \sum \frac{q_i}{r_i}$$

$$= k \left(\frac{-e}{a} + \frac{e}{2a} \right)$$

$$= -\frac{ke}{2a} < 0$$

$$V(C) = V(A) = -\frac{ke}{2a}$$

$$V(B) = k \left(\frac{+e}{a} + \frac{e}{a} \right) = \frac{2ke}{a} > 0$$

$$3. E_{pe}(A) = q(A) \times V(A) = e \times -\frac{ke}{2a} = -\frac{ke^2}{2a} < 0$$

$$E_{pe}(C) = E_{pe}(A) = -\frac{ke^2}{2a} < 0$$

$$E_{pe}(B) = q(B) \times V(B) = -e \times \frac{2ke}{a} = -\frac{2ke^2}{a} < 0$$

Exercice 3

$$1. \vec{E} = -\text{grad}(V)$$

$V(r)$ ne dépend que de $r \Rightarrow \vec{E}$ n'a qu'une seule composante $E = E_r$, $\vec{E} = \vec{E}_r$, on dit que \vec{E} est radial.

$$\vec{\text{grad}} = \begin{pmatrix} \frac{\partial}{\partial r} \\ \frac{1}{r} \frac{\partial}{\partial \theta} \\ \frac{1}{r \sin \theta} \frac{\partial}{\partial \varphi} \end{pmatrix} \quad \begin{aligned} E_r &= -\frac{\partial}{\partial r} \\ E_\theta &= \frac{1}{r} \frac{\partial}{\partial \theta} = 0 \\ E_\varphi &= \frac{1}{r \sin \theta} \frac{\partial}{\partial \varphi} = 0 \end{aligned}$$

$$2. E_r = -\frac{\partial V}{\partial r} = -\frac{dV}{dr} = -V'(r)$$

$$V(r) = \frac{q}{4\pi\epsilon_0} \times \frac{1}{r} \left(e^{-\frac{r}{a_0}} \right)$$

$$u = \frac{1}{r} \Rightarrow u' = -\frac{1}{r^2} \quad \text{et } v = e^{-\frac{r}{a_0}} \Rightarrow v' = -\frac{1}{a_0} e^{-\frac{r}{a_0}}$$

$$\text{Donc } E_r = -V'(r)$$

$$= -\frac{q}{4\pi\epsilon_0} \left(-\frac{1}{r^2} e^{-\frac{r}{a_0}} + \left(-\frac{1}{a_0}\right) e^{-\frac{r}{a_0}} \right)$$

$$= \frac{q}{4\pi\epsilon_0} e^{-\frac{r}{a_0}} \left(\frac{1}{r^2} + \frac{1}{a_0} \right)$$

$$E_r(r) = \frac{q}{4\pi\epsilon_0} \frac{e^{-\frac{r}{a_0}}}{r^2} \left(1 + \frac{r}{a_0} \right)$$