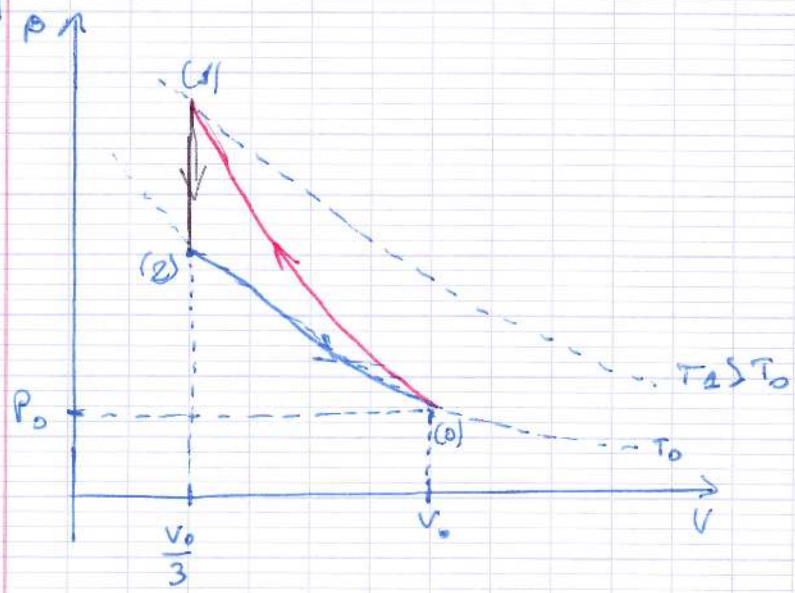
Physique
9/04

①

Série 12 : Gaz Parfait - Premier principe - Cycles.

Exercice n° 1 : 1)



2) Calcul T_1, P_1, P_2 en fct de T_0, P_0, γ.

$PV^\gamma = cste$.

$\frac{nRT}{V} \cdot V^\gamma = cste \iff T \cdot V^{\gamma-1} = cste$ car nR cstes.

$0 \to 1$.

$T_0 V_0^{\gamma-1} = T_1 V_1^{\gamma-1} \iff T_0 V_0^{\gamma-1} = T_1 \left(\frac{V_0}{3}\right)^{\gamma-1}$

$\iff \boxed{T_1 = 3^{\gamma-1} \times T_0}$

Calcul de P_1 :

à l'état $P_1 V_1 = RT_1 \Rightarrow P_1 \times \frac{V_0}{3} = RT_0 \cdot 3^{\gamma-1}$

$P_1 = \frac{RT_0}{V_0} \times 3^\gamma \Rightarrow P_1 = P_0 3^\gamma$

Calcul de P_2 :

$P_2 \cdot \frac{V_0}{3} = R \cdot T_0 \iff P_2 = \frac{3RT_0}{V_0} \Rightarrow P_2 = 3P_0$

Exercice n°2 : 3)

$$\begin{cases} T_1 = T_0\, 3^{\gamma-1} \\ P_1 = P_0 \cdot 3^{\gamma} \\ P_2 = 3P_0 \end{cases}$$

Transf	$W = -\int_{V_i}^{V_f} P\,dV$	$Q = \Delta U - W$	$\Delta U = n c_v \Delta T$ $(n=1)$
$0 \to 1$ adiabatique (compression)	$W = \Delta U$ car $Q=0$ $W_{0\to1} = c_v T_0 (3^{\gamma-1} - 1)$	$Q_{0\to1} = 0$	$\Delta U = c_v (T_1 - T_0)$ $= c_v T_0 (3^{\gamma-1} - 1)$.
$1 \to 2$ isochore	$W_{1\to2} = 0$	$Q = \Delta U$ $= c_v T_0 (1 - 3^{\gamma-1})$	$\Delta U_{1\to2} = c_v (T_0 - T_1) = -c_v T_0 (1 - 3^{\gamma-1})$ $= -\Delta U_{0\to1}$
$2 \to 0$ isotherme (detente)	$W_{2\to0} = -RT_0 \ln\left(\frac{V_0}{\frac{V_0}{3}}\right)$ $W_{2\to0} = -RT_0 \ln(3) < 0$	$Q = -W_{2\to0}$ $Q = RT_0 \ln(3)$	$\Delta U_{2\to0} = 0$

$$W_{cycle} = W_{0\to1} + W_{2\to0} = c_v T_0 (3^{\gamma-1} - 1) - RT_0 \ln(3) = RT_0 \left(\frac{3^{\gamma-1} - 1}{\gamma - 1} - \ln(3)\right)$$

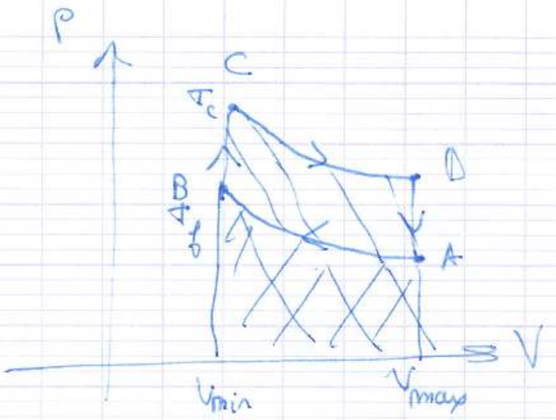
avec $\left(c_v = \frac{R}{\gamma-1}\right)$

$W_{0\to1} > 0$ (car compression $V \searrow \Rightarrow dv < 0 \Rightarrow -P\,dV > 0$)

$W_{2\to0} < 0$ (car detente $V \nearrow \Rightarrow dv > 0 \Rightarrow -P\,dV < 0$)

$W_{0\to1} > |W_{2\to0}|$ en comparant les 2 aires sous les courbes adiabatiques et isotherme.

Exercice n° 2 :



T_c détente $\underset{C \to D}{}$ $W_{CD} < 0$

T_f compression $\underset{A \to B}{}$ $W_{AB} > 0$

L'aire sous l'isotherme T_c est plus importante que celle de l'isotherme T_f

d'où $|W_{CD}| > W_{AB}$

$\Rightarrow W_{CD} + W_{AB} < 0 \Rightarrow W_{cycle} < 0 \Rightarrow$ cycle moteur.

2a) • $Q_{BC} = \Delta U_{BC}$ car $W_{BC} = 0$

$Q_{BC} = n c_v (T_C - T_B) = n c_v (T_c - T_f) > 0$

• $Q_{DA} = \Delta U_{DA}$ $W_{DA} = 0$

$Q_{DA} = n c_v (T_A - T_D) = n c_v (T_f - T_c) < 0$

$\boxed{\underbrace{Q_{DA}}_{\text{cédée}} = - \underbrace{Q_{BC}}_{\text{absorbé}}}$

Transf	$W = -\int_{V_i}^{V_f} P dV$	$Q = \Delta U - W$	$\Delta U = n c_v \Delta T$ GP
B → C isoch	$W_{BC} = 0$	$Q_{BC} = \Delta U = n c_v (T_c - T_f)$ (>0)	$\Delta U_{BC} = n c_v (T_c - T_f)$
C → D isoth	$W_{CD} = -nRT_c \ln\left(\frac{V_{max}}{V_{min}}\right)$ (<0)	$Q_{CD} = nRT_c \ln\left(\frac{V_{max}}{V_{min}}\right)$ $Q_{CD} = -W_{CD} > 0$	$\Delta U_{C \to D} = 0$
D → A isoch	$W_{DA} = 0$	$Q_{DA} = \Delta U = n c_v (T_f - T_c)$ (<0)	$\Delta U_{D \to A} = n c_v (T_f - T_c)$
A → B isoth	$W_{A \to B} = -nRT_f \ln\left(\frac{V_{min}}{V_{max}}\right)$	$Q_{AB} = nRT_f \ln\left(\frac{V_{min}}{V_{max}}\right)$ $Q_{AB} = -W_{AB}$ car $\Delta U = 0$	$\Delta U_{AB} = 0$

Le rendement $r = \dfrac{\text{Energie utile (cédée ou fournie)}}{E_{\text{absorbée}} \text{ (consommée)}}$

Dans notre cas

$$r = \frac{|W_{cycle}|}{Q_{CD}} = \frac{|W_{CD} + W_{AB}|}{Q_{CD}}$$

$$Q_{CD} > 0$$

$$r = \frac{\left|-\cancel{n}\cancel{R}T_c \cancel{\ln\left(\frac{V_{max}}{V_{min}}\right)} + \cancel{n}\cancel{R}T_f \cancel{\ln\left(\frac{V_{max}}{V_{min}}\right)}\right|}{\cancel{n}\cancel{R}T_c \cancel{\ln\left(\frac{V_{max}}{V_{min}}\right)}} = \frac{|-T_c + T_f|}{T_c} = \frac{T_c - T_f}{T_c} =$$

$$r = 1 - \frac{T_f}{T_c}$$