Régime variable

$v' =$

de 0 à αT

$v' = U'_{max}$

de αT à T

$v' = U'_{min}$

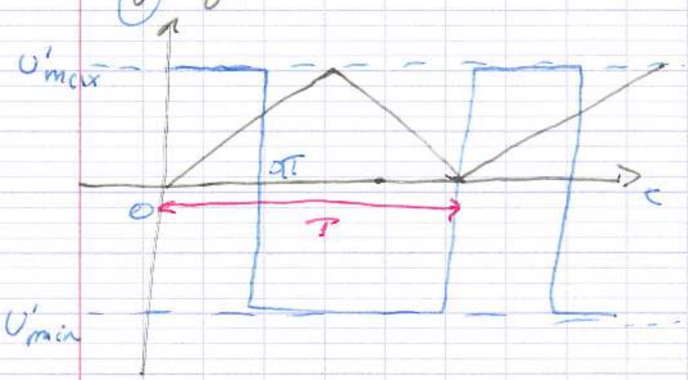
v''

de 0 à $T/2$:

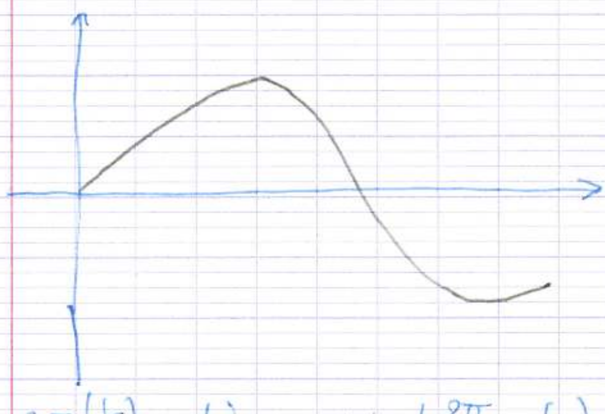
$v''(t) = \frac{2 . U'_{max}}{T} . t$

de $T/2$ à T

$v''(t) = -2 . \frac{U'_{max} t}{T} + 2 U'_{max}$



Régime sinusoïdale



$v(t) = U_{max} . \sin\left(\frac{2\pi}{T} . t\right)$

ou

$v(t) = U_{max} . \cos\left(\frac{2\pi}{T} . t - \frac{\pi}{2}\right)$

Cas général :

$e(t) = E_{max} . \sin\left(\frac{2\pi}{T} t - \varphi\right)$

Regime variable S2
TD1 : Régimes variables

Valeur moyenne de $v(t)$

$$\langle v \rangle = \frac{1}{T}\int_{\theta}^{\theta+T} v(t)\,dt$$

1) $\langle v \rangle = \frac{1}{T}\int_{0}^{T} v(t)\,dt$

$v(t)$: de 0 à aT : $v(t) = V_M$

$v(t)$: de aT à T : $v(t) = 0$

$$v(t) = \frac{1}{T}\int_{0}^{aT} v(t)\,dt + \frac{1}{T}\int_{aT}^{T} 0\,dt$$

$$= \frac{1}{T}\int_{0}^{aT} V_M\,dt + \frac{1}{T}\int_{aT}^{T} 0\,dt$$

$$= \frac{1}{T}\left[V_M t\right]_{0}^{aT}$$

$$= \frac{1}{T}(aT V_M - 0)$$

$$\langle v(t) \rangle = aV_M$$

Valeur efficace :

$$v(t) = \frac{1}{T}\int_{0}^{aT} V_M^2\,dt$$

$$= \frac{1}{T}\left[V_M^2 t\right]_{0}^{aT}$$

$$V^2 = \frac{1}{T} V_M^2\, aT = V_M^2 a$$

$$V = \sqrt{a}\, V_M$$

Valeur moyenne

2) $\langle v \rangle = \frac{1}{T}\int_{0}^{T} v(t)\,dt$

$$= \frac{1}{T}\int_{0}^{2T/3} v(t)\,dt + \frac{1}{T}\int_{2T/3}^{T} v(t)\,dt$$

$$= \frac{1}{T}\left[10t\right]_{0}^{2T/3} + \frac{1}{T}\left[-5t\right]_{2T/3}^{T}$$

$$= \frac{1}{T} \times 10 \times \frac{2T}{3} + \frac{1}{T}\left[-5T - \left(-5 \times \frac{2T}{3}\right)\right]$$

$$= \frac{20}{3} - 5 + \frac{10}{3}$$

$$= 10 - 5 = 5\,V$$

$$v^2(t) = \frac{1}{T}\int_{0}^{T} v^2(t)\,dt$$

$$= \frac{1}{T}\int_{0}^{\frac{2T}{3}} 10^2\,dt + \frac{1}{T}\int_{2T/3}^{T} (-5)^2\,dt$$

$$= \frac{1}{T}\left[100t\right]_{0}^{2T/3} + \frac{1}{T}\left[25t\right]_{2T/3}^{T}$$

$$= \frac{1}{T} \times \frac{200T}{3} + \frac{1}{T}\left(25T - 25 \times \frac{2T}{3}\right)$$

$$= \frac{200}{3} + 25 - \frac{50}{3}$$

$$= \frac{200}{3} + \frac{25}{3} \Rightarrow V = \sqrt{\frac{225}{3}} = \sqrt{75}\,V = 5\sqrt{3}\,V$$

3) $\langle v(t)\rangle = \frac{1}{T}\int_0^T v(t)\,dt = \frac{1}{T}\int_0^{T/2} \underbrace{v(t)}_{V_M}\,dt + \frac{1}{T}\int_{T/2}^{T} \underbrace{v(t)}_{-V_M}\,dt$

$v(t)$ de 0 à $\frac{T}{2}$ $\qquad = \frac{1}{T}\left[V_M t\right]_0^{T/2} + \frac{1}{T}\left[-V_M t\right]_{T/2}^{T}$

$\Rightarrow v(t) = \frac{4V_M t}{T} - V_{Max}$

$v(t)$: de $\frac{T}{2}$ à T

$\Rightarrow v(t) = \frac{-4V_M t}{T} + V_{Max}$

valeur efficace :

$$V = \frac{1}{T}\int_0^{T/2}\left(\frac{V_M t}{T} - V_{Max}\right)^2 dt + \frac{1}{T}\int_{T/2}^{T}\left(\right.$$

c) $V^2 = \frac{2}{T}\int_0^{T/2} v^2(t)\,dt$

$V^2 = \frac{2}{T}\int_0^{T/2} \left(-v_M + \frac{4v_M(t)}{T}\right)^2 dt$

$V^2 = \frac{2}{T}\int_0^{T/2} \left(v_M^2 - \frac{8v_M^2 t}{T} + \frac{16v_M^2 t^2}{T^2}\right) dt$

$V^2 = v_M^2 - \frac{2}{T}\left[\frac{4v_M^2 t^2}{T}\right]_0^{T/2} + \frac{2}{T}\left[\frac{16v_M^2 t^3}{3T^2}\right]_0^{T/2}$

$V^2 = V_M^2 - \frac{2}{T} \times \frac{4V_M^2}{T} \times \frac{T^2}{2^2} + \frac{2}{T} \times \frac{16V_M^2}{3T^2} \times \frac{T^3}{2^3}$

$V^2 = V_M^2 - 2V_M^2 + \frac{4V_M^2}{3}$

$V^2 = \frac{3V_M^2 - 6V_M^2 + 4V_M^2}{3}$

$V^2 = \frac{V_M^2(3-6+4)}{3}$

$V^2 = \frac{V_M^2}{3}$

d) de 0 à $T/3$: $v(t) = V_M$

de $T/3$ à T : $v(t) = \frac{-3V_M t}{2T} + \frac{3}{2}V_M$

coef dir : $m = \frac{0 - V_M}{\frac{2T}{3}} = \frac{-3V_M}{2T}$

à $\frac{T}{3}$: $v\left(\frac{T}{3}\right) = V_M = \frac{-3V_M}{2T} \times \frac{T}{3} + b$

$b = V_M + \frac{3V_M}{6}$

$= \frac{9V_M}{6}$

$= \frac{3}{2}V_M$

$$\langle v\rangle = \frac{1}{T}\int_0^{T/3} V_M\,dt + \frac{1}{T}\int_{T/3}^{T}\left(\frac{-3V_M t}{2T} + \frac{3}{2}V_M\right)dt$$

$$= \frac{V_M}{3} - \frac{3V_M}{2T^2}\int_0^{T/3} t\,dt + \frac{3V_M}{2T}\int_{T/3}^{T} 1.dt$$

$$= \frac{V_M}{3} - \frac{3V_M}{2T^2}\left(\frac{T^2}{2} - \frac{T^2}{18}\right) + \frac{3V_M}{2T}\left(T - \frac{T}{3}\right)$$

$$\langle v\rangle = \frac{V_M}{3} - \frac{3V_M}{2\cancel{T^2}}\left(\frac{8\cancel{T^2}}{18}\right) + \frac{\cancel{3}V_M}{\cancel{2T}}\left(\frac{\cancel{2T}}{\cancel{3}}\right)$$

$$= \frac{V_M}{3} - \frac{24V_M}{36} + V_M$$

$$= \frac{12V_M}{36} - \frac{24V_M}{36} + \frac{36V_M}{36}$$

$$= \frac{V_M(12-24+36)}{36} = \frac{24V_M}{36} = \frac{2V_M}{3}$$

$$V^2 = \frac{1}{T}\int_0^T v(t)\,dt$$

$$\begin{cases} 0 \to T/3 : v(t) = V_M \\ T/3 \to T : v(t) = \frac{-3V_M}{2T}t + \frac{3}{2}V_M \end{cases}$$

$$V^2 = \frac{1}{T}\int_0^{T/3} V_M^2\,dt + \int_{T/3}^{T} \frac{9V_M^2}{4T^2} - 6 \ldots$$

$$\begin{cases} 0 \to T/3 : v^2(t) = V_M^2 \\ T/3 \to T : v^2(t) = \left(\frac{-3V_M t}{2T} + \frac{3V_M}{2}\right)^2 \end{cases}$$

$$= \frac{9V_M^2 t^2}{4T^2} + \frac{-6V_M t}{2T} \times \frac{3V_M}{2} + 9\ldots$$

$$V^2 = \frac{1}{T}\int_0^{T/3} V_M^2\,dt + \frac{1}{T}\int_{T/3}^{T} \frac{9V_M^2}{4T^2}t^2\,dt + \frac{1}{T}\int_{T/3}^{T} \frac{-18V_M^2 t}{4T} + \frac{1}{T}\int_{T/3}^{T} \frac{9V_M^2}{4}$$

$$V^2 = \frac{V_M^2}{3} + \frac{9V_M^2}{4T^3}\left[\frac{t^3}{3}\right]_{T/3}^{T} - \frac{18V_M^2}{4T^2}\left[\frac{t^2}{2}\right]_{T/3}^{T} + \frac{9V_M^2}{4T}\cdot\frac{2T}{3}$$

$$V^2 = \frac{V_M^2}{3} + \frac{9V_M^2}{4T^3}\left(\frac{T^3}{3} - \frac{T^3}{81}\right) - \frac{18V_M^2}{4T^2}\left(\frac{T^2}{2} - \frac{T^2}{18}\right) + \frac{9V_M^2}{4\cancel{T}}\cdot\frac{2\cancel{T}}{3}$$

$$V^2 = \frac{V_M^2}{3} + \frac{\cancel{9}V_M^2}{4\cancel{T^3}}\left(\frac{26T^3}{81}\right) - \frac{\cancel{18}V_M^2}{4\cancel{T^2}}\left(\frac{\cancel{8T^2}}{\cancel{18}}\right) + \frac{3V_M^2}{2}$$

$$V^2 = \frac{V_M^2}{3} + \frac{13V_M^2}{18} - 2V_M^2 + \frac{3V_M^2}{2}$$

$$V^2 = \frac{5V_M^2}{9} \quad \frac{10V_M^2}{18} \quad \frac{6V_M^2 + 13V_M^2 - 36V_M^2 + 27V_M^2}{18}$$

$$\frac{9T^2}{18} - \frac{T^2}{18} = \frac{8T^2}{18}$$

-6

$$\frac{-6V_M t}{2T} + \frac{3V_M}{2}$$

$$\frac{27T^3}{81} - \frac{T^3}{81}$$

$$\frac{26T^3}{81}$$

$$\frac{V_M^2}{2}\cdot\frac{13}{9}$$

$$\frac{13V_M^2}{18}$$

$$\frac{6V_M^2}{18} + \frac{13V_M^2}{18}$$

$$\frac{19V_M^2}{18}$$

5) de $0 \to T/2$: $v(t) = V_M \cdot \sin(\frac{2\pi}{T} \cdot t)$

$T/2 \to T$: $v(t) = 0$

$$\langle v \rangle = \frac{1}{T} \int_{T/2}^{T} \sin\left(\frac{2\pi}{T} . t\right) dt$$

$$\langle v \rangle = \frac{V_M}{T} \left[-\cos\left(\frac{2\pi}{T} . t\right) * \frac{T}{2\pi} \right]_0^{T/2}$$

$$\langle v \rangle = \frac{V_M}{T} \times \frac{T}{2\pi} \left[-\cos\left(\frac{2\pi}{T} . t\right) \right]_0^{T/2}$$

$$\langle v \rangle = \frac{V_M}{2\pi} \left[1 - (-1) \right]$$

$$\langle v \rangle = \frac{V_M}{\pi}$$

V^2 $\quad 0 \to T/2 = (V_M^2 \sin^2(\frac{2\pi}{T} . t)$

$$V^2 = \frac{1}{T} \int_0^{T/2} V_M^2 \sin^2\left(\frac{2\pi}{T} . t\right) dt$$

$$V^2 = \frac{V_M^2}{T} \int_0^{T/2} \sin^2\left(\frac{2\pi}{T} . t\right) dt$$

$$V^2 = \frac{V_M^2}{T} \int_0^{T/2} \sin^2(\omega t)\, dt = \frac{V_M^2}{T} \int_0^{T/2} \sin^2(2\pi f t)\, dt$$

$$\cos(2a) = \cos^2(a) - \sin^2(a)$$
$$= 1 - 2\sin^2(a)$$
$$\sin^2 a = \frac{1 - \cos(2a)}{2}$$

$$V^2 = \frac{V_M^2}{T} \int \frac{1 - \cos(4\pi f t)}{2} dt$$

$$V^2 = \frac{V_M^2}{T} \left[\frac{1 - \cos(2 \cdot \frac{2\pi}{T} t)}{2} dt \right]_0^{T/2} = \frac{V_M^2}{T} \left(\frac{1 - \cos\left(\frac{4\pi}{T} \times \frac{T}{2}\right)}{2} \right)$$

$$V^2 = \frac{V_M}{2}$$